Effect of polarized gluon distribution on π^0 spin asymmetry

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(Dated: November 9, 2004)

A longitudinal double spin asymmetry for π^0 production has been measured by the PHENIX collaboration. The preliminary data indicate the negative asymmetry at transverse momentum $p_T = 1 \sim 5$ GeV, whereas theoretical predictions using perturbative QCD are positive asymmetry. We study effects of polarized gluon distribution on the spin asymmetry and suggest the possibility to obtain a sizable negative asymmetry in larger p_T region.

PACS numbers: 13.85.Ni, 13.88.+e

Determination of the polarized parton distribution functions (PDFs) is crucial for understanding the spin structure of the nucleon [1]. As is well known, the proton spin is composed of the spin and angular momentum of quarks and gluons. Several parametrizations of the polarized PDFs have been proposed, and have successfully reproduced experimental data [2, 3, 4, 5, 6]. In particular, the amount of the proton spin carried by quarks is determined well by a global analysis with the polarized deep inelastic scattering (DIS) data. The value is about $\Delta\Sigma = 0.1 \sim 0.3$, whereas the prediction from the naive quark model is $\Delta\Sigma = 1$. This surprising result leads to extensive study on the gluon polarization. The current parametrizations suggest a large positive polarization of gluon. However, our knowledge about the polarized gluon distribution $\Delta g(x,Q^2)$ is still poor, since theoretical and experimental uncertainties are rather large. The determination of $\Delta g(x,Q^2)$ gives us a clue to the proton spin puzzle.

The RHIC is the first high energy polarized proton-proton collider to measure $\Delta g(x,Q^2)$ [7]. We can extract information about $\Delta g(x,Q^2)$ through various processes, e.g., prompt photon production, jet production, and heavy flavor production. These processes are quite sensitive to $\Delta g(x,Q^2)$, since gluons in the initial state associate with the cross section in leading order (LO).

Recently, the PHENIX collaboration has reported preliminary results [8] for inclusive π^0 production $pp \to \pi^0 X$ which is also likely to be sensitive to $\Delta g(x,Q^2)$. The double spin asymmetry was measured in longitudinally polarized proton-proton collisions at RHIC in the kinematical ranges: center-of-mass (c.m.) energy $\sqrt{s} = 200$ GeV and central rapidity $|\eta| \le 0.38$. The data suggest that the asymmetry is significantly negative at transverse momentum $p_T = 1 \sim 5$ GeV, whereas there is no theoretical predictions indicating such an asymmetry.

In this letter, we study the ambiguity of π^0 double spin asymmetry stemming from the $\Delta g(x,Q^2)$ uncertainty. We prepare three different functional forms of $\Delta g(x,Q^2)$. The possibility to derive the negative asymmetry at moderate p_T is demonstrated by using the modified $\Delta g(x,Q^2)$. Furthermore, we suggest that the asymmetry in larger p_T region is more sensitive to the functional form of $\Delta g(x,Q^2)$.

We have calculated the longitudinal double spin asymmetry which is defined by

$$A_{LL}^{\pi^0} \equiv \frac{[d\sigma_{++} - d\sigma_{+-}]/dp_T}{[d\sigma_{++} + d\sigma_{+-}]/dp_T} = \frac{d\Delta\sigma/dp_T}{d\sigma/dp_T}, \qquad (1)$$

where p_T is the transverse momentum of produced pion. $d\sigma_{hh'}$ denotes the spin-dependent cross section with definite helicity h and h' for incident protons.

The cross sections can be separated short distance parts from long distance parts in the QCD factorization theorem. The short distance parts represent interaction amplitudes of hard partons, and are calculable in the framework of perturbative QCD (pQCD). On the other hand, the long distance parts such as PDFs should be determined by using experimental data. The polarized cross section $\Delta\sigma$ is written in terms of the polarized PDFs $\Delta f_i(x,Q^2)$ as follows:

$$\begin{aligned}\frac{d\Delta\sigma^{pp\to\pi^0 X}}{dp_T} &= \sum_{a,b,c}\int_{\eta^{\min}}^{\eta^{\max}} d\eta \int_{x_a^{\min}}^{1} dx_a \int_{x_b^{\min}}^{1} dx_b \\ &\times \Delta f_a(x_a,Q^2)\Delta f_b(x_b,Q^2) \\ &\times \mathcal{J}\left(\frac{\partial(\hat{t},z)}{\partial(p_T,\eta)}\right)\frac{\Delta\hat{\sigma}^{ab\to cX}(\hat{s},\hat{t})}{d\hat{t}} \\ &\times D_c^{\pi^0}(z,Q^2), \end{aligned}\qquad (2)$$

where the sum is over the partonic processes $a+b \to c+X$ associated with π^0 production. $\mathcal{J}$ is the Jacobian which transforms kinematical variables from $\hat{t}$ and z into p_T and η of the produced π^0. $\Delta\hat{\sigma}$ describes the polarized cross section of subprocesses. The partonic Mandelstam variables $\hat{s}$ and $\hat{t}$ are defined by $\hat{s} = (p_a + p_b)^2$ and $\hat{t} = (p_a - p_c)^2$ with partonic momentum p_i, respectively. The squared c.m. energy s which is related to $\hat{s}$ through $\hat{s} = x_a x_b s$ and the pseudo-rapidity η are set as $\sqrt{s} = 200$ GeV and $|\eta| \le 0.38$ in the PHENIX acceptance. $D_c^{\pi^0}(z,Q^2)$ represents the spin-independent fragmentation function decaying into pion $c \to \pi^0$ with a momentum fraction z.

In this analysis, the cross sections and the spin asymmetry are calculated in LO level. Rigorous analysis of $\mathcal{O}(\alpha_s^3)$ next-to-leading order (NLO) calculation has been established in Ref. [9]. We consider that the qualitative behavior of the asymmetry does not change, even if NLO corrections are included in our study. In numerical calculations, we adopt the AAC set [2] as the polarized PDFs and the KKP set [12] as fragmentation functions. We choose the scale $Q^2 = p_T^2$.

The partonic subprocesses in LO are composed of $\mathcal{O}(\alpha_s^2)$ $2 \to 2$ tree-level channels listed as $gg \to q(g)X$, $qg \to q(g)X$, $qq \to qX$, $q\bar{q} \to q(g,q')X$, $qq' \to qX$, and $q\bar{q}' \to qX$ including channels of the permutation $q \leftrightarrow \bar{q}$. Main contribution to the polarized cross section comes from $gg \to q(g)X$ and $qg \to q(g)X$ channels with conventional PDFs. The gg contribution dominates in low p_T region and steeply decreases with p_T. Then, the qg process becomes dominant contribution in larger p_T region. The crossing point of these processes however depends on parametrization of the polarized PDFs. In both cases, the spin asymmetry for π^0 production is sensitive to the gluon polarization.

As mentioned above, the partonic cross section $\Delta\hat{\sigma}$ is well-defined in the pQCD framework. Hence, as a cause of inconsistency with the PHENIX data, we consider the ambiguity of long distance parts: fragmentation functions and PDFs.

The fragmentation into π^0 includes all channels $q, \bar{q}, g \to \pi^0$. Each component of $D_c^{\pi^0}$ would be determined by a global analysis [11, 12] of several experiments. In addition, one can obtain information about $D_c^{\pi^0}$ from π^0 production in unpolarized pp collisions at $\sqrt{s} = 200$ GeV at RHIC as well. Actually, the unpolarized cross section measured by the PHENIX [13] are consistent with NLO pQCD calculations with good accuracy. Therefore, the asymmetry is not strongly affected by $D_c^{\pi^0}$, even though $D_c^{\pi^0}$ has some uncertainties.

In the polarized reaction that we discuss, kinematical ranges and fragmentation functions are the same as the unpolarized case except the polarized PDFs. For the polarized quark distributions $\Delta q(x)$ and $\Delta\bar{q}(x)$, the antiquark distributions and their flavor structure are not well known. For π^0 production, subprocesses are (light quark) flavor blind reaction, and the predominant qg process depends on the sum $\Delta q(x)+\Delta\bar{q}(x)$ which is well determined in the polarized DIS. Therefore, we can neglect effects of ambiguities of the quark polarization on the asymmetry. This fact implies that we have large ambiguity arising from the polarized gluon distribution $\Delta g(x)$.

For investigating the effect of $\Delta g(x)$ on the asymmetry, we prepare three functional forms as shown in Fig 1. Solid curve shows $\Delta g(x)$ by the global analysis with the polarized DIS data [2]. Dashed and dot-dashed curves show two artificial modified $\Delta g(x)$, respectively. The sample-1 distribution has a node, and the sample-2 distribution is small negative polarization. Since the sample-1 and 2 are within the $\Delta g(x)$ uncertainty by the AAC analysis, these distributions can be adopted as a model of $\Delta g(x)$. These are taken account of the Q^2 dependence by the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equation with the polarized quark and antiquark distributions.

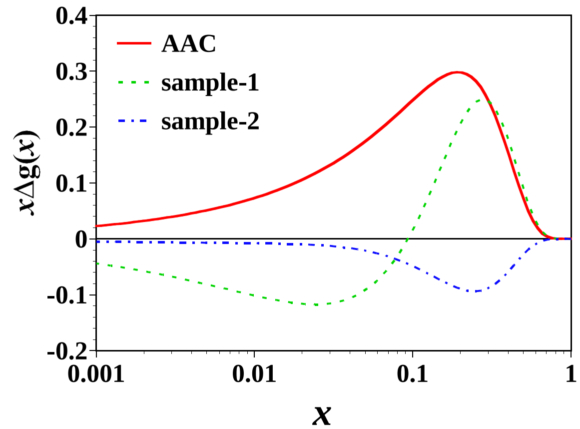

FIG. 1: Polarized gluon distributions $\Delta g(x)$ at $p_T = 2.5$ GeV. Solid, dashed, and dot-dashed curves indicate the AAC, sample-1, and 2 distributions, respectively.

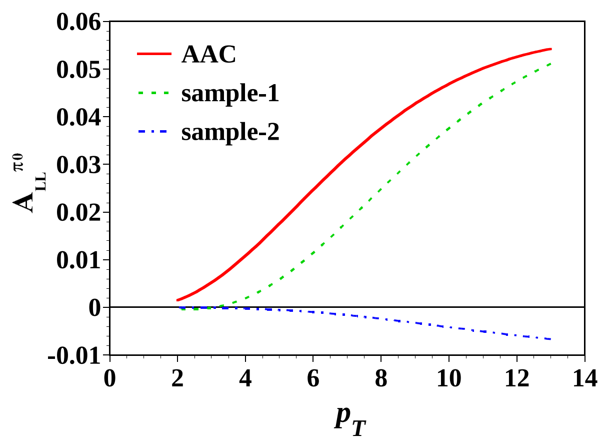

FIG. 2: Spin asymmetries for π^0 production by using three different $\Delta g(x)$ in Fig. 1.

We discuss behavior of the spin asymmetry associated with the functional form of $\Delta g(x)$. The obtained asymmetries with these gluon distributions are shown in Fig. 2. We find that the asymmetry for the AAC $\Delta g(x)$ is positive in whole p_T region. The asymmetries for the sample-1 and 2 become negative at low p_T. In particular, we obtained the negative asymmetry in whole p_T region by using the sample-2 $\Delta g(x)$. Further, there are variations of these asymmetries at large p_T.

The asymmetry for the AAC is positive and on the increase with p_T. The positive polarization for $\Delta g(x)$ brings about positive contributions for gg and qg processes, which are dominant contributions to the asymmetry. In this case, the asymmetry cannot become negative.

The positive $\Delta g(x)$ is suggested by the recent global analyses with the polarized DIS data [2, 3, 4, 5, 6]. However, the $\Delta g(x)$ could not determined well and has large uncertainty, although these analyses obtain good agreement with the experimental data. At this stage, we cannot rule out the negative polarization for $\Delta g(x)$. Therefore, there is a possibility of the negative asymmetry with the modified $\Delta g(x)$.

For the sample-1 in Fig. 2, the asymmetry is slight negative in low p_T and changes into positive at $p_T = 3$ GeV. In the region $p_T < 3$ GeV, we find that the gg and qg processes are negative contributions, respectively. The negative contribution for gg process would be needed opposite polarizations of $\Delta g(x)$ at x_a and x_b. Computed by using several configurations of $\Delta g(x)$ with a node, the gg contribution is not always negative. The contribution basically depends on the shape of $\Delta g(x)$ even if it has a node. For instance, the $\Delta g(x)$ needs rapid changing polarization at the node as shown in Fig. 1, because $|x_a - x_b|$ dose not become larger in the PHENIX acceptance $|\eta| \leq 0.38$. The slight negative asymmetry with the $\Delta g(x)$ having a node is suggested in Ref. [10]. Such a functional form has the possibility of making the small negative asymmetry at low p_T.

In the region $p_T > 3$ GeV, the gg contribution changes positive, and dominates in the region $p_T < 10$ GeV. This is because that the node rapidly shifts toward low x direction due to Q^2 evolution with increasing p_T. Therefore, the positive polarization for $\Delta g(x)$ at medium x contributes predominantly to the positive asymmetry via the gg process. The asymmetry at large p_T is sensitive to the behavior of $\Delta g(x)$ at medium x.

As another possibility of the negative asymmetry, we choose slight negative polarization for $\Delta g(x)$. In this case, the gg contribution is positive while the qg contribution is negative. The asymmetry is determined by a difference between two contributions. The gg and qg contributions are proportional to $(\Delta g)^2$ and Δg, respectively. The gg contribution is more sensitive to behavior of $\Delta g(x)$. In particular, the $\Delta g(x)$ at low x significantly affects on the contribution at low p_T since the value of $x^{\min}$ in Eq. 2 is rather small. In order to make the positive gg contribution smaller, the $\Delta g(x)$ for the sample-2 is taken small polarization at low x as shown in Fig. 1.

In Fig. 2, as far as the sample-2 is concerned, the asymmetry indeed becomes negative in whole p_T region. In the region $p_T < 3$ GeV, the small negative polarization for $\Delta g(x)$ promotes slight positive contribution for the gg process. In this case, the gg contribution is the same order of magnitude as the qg contribution, and almost cancel out the negative contribution. The asymmetry therefore is determined by other processes except above two processes. The total contribution of the processes becomes slight negative. Above the region, the gg contribution rapidly decrease with increasing p_T. The qg process becomes dominant contribution, which provides the negative asymmetry. Thus, the negative asymmetry in whole p_T region can be obtained by using the negative $\Delta g(x)$ which makes the qg contribution larger than the gg contribution.

In the sample-2, we should note that the magnitude of $\Delta g(x)$ at the minimum point is not too large. This is because that the shape of $\Delta g(x)$ is rapidly varied by the Q^2 evolution, the minimum point of $\Delta g(x)$ shifts toward lower x and the width broadens. At moderate p_T, the gg process is more sensitive to the low-x behavior of the evolved $\Delta g(x)$ than the qg process. If the $\Delta g(x)$ is taken large negative polarization at the minimum point, the magnitude of the gg contribution becomes rapidly larger than that of the qg contribution, and then the asymmetry becomes positive at moderate p_T. The small negative $\Delta g(x)$ therefore is required to obtain the negative asymmetry in whole p_T region.

In above cases, there is no way to derive large negative value for the asymmetry of a few percent levels. In this study, slight negative asymmetries can be obtained at low p_T where the preliminary PHENIX data exist. Attempting to make large negative asymmetry by using several different shapes of $\Delta g(x)$, we cannot obtain negative value above 0.1% in the region $p_T < 3$ GeV. Further, even if the asymmetry is positive, the magnitude is below 1% in the region. Although the polarized PDFs are controllable, the absolute value of the asymmetry does not become so large at low p_T. As discussed above, the functional form of $\Delta g(x)$ needs some restraints to make the asymmetry negative. It is difficult to obtain sizable negative value in comparison with the positive case.

In large p_T, the difference of the obtained asymmetries remarkably reflects the medium-x behavior of $\Delta g(x)$. Experimental data in the region have useful information for the $\Delta g(x)$ determination. For instance, the asymmetry for the sample-2 becomes rather larger to negative direction. If future precise data approve of the negative asymmetry in the region, the $\Delta g(x)$ requires significant modification of its functional form and has the possibility of negative polarization. It has the potential of the negative gluon contribution to the nucleon spin. Therefore, clarifying the gluon contribution is needed data constraint in wide p_T range.

Next, we consider the effect of the π^0 data on the $\Delta g(x)$ determination in terms of the uncertainty estimation for the spin asymmetry. The large uncertainty of $\Delta g(x)$ implies the difficulty of extracting the gluon contribution from the polarized DIS data. We therefore have interest in the effect on the $\Delta g(x)$ uncertainty if the experimental data are included in the global analysis. As simple evaluation, we compare the asymmetry uncertainty with the experimental errors by the PHENIX.

The asymmetry uncertainty coming from the polarized PDFs is defined as the ratio of the polarized cross section uncertainty and the unpolarized cross section: $\delta A_{LL}^{\pi^0} = \delta\Delta\sigma^{\pi^0}/\sigma^{\pi^0}$. The cross section uncertainty is estimated

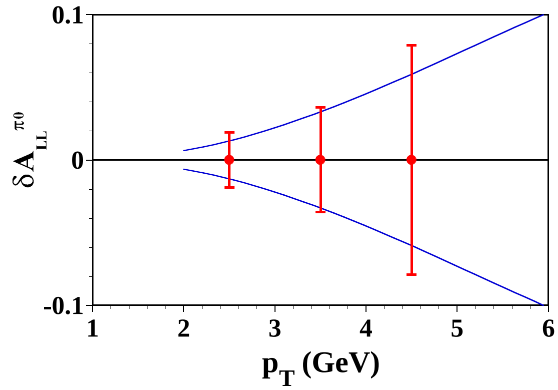

FIG. 3: Comparison the asymmetry uncertainty $\delta A_{LL}^{\pi^0}$ with the experimental errors for $\sqrt{s} = 200$ GeV.

by the Hessian method, and is given by

$$\left[\delta\Delta\sigma^{\pi^0}\right]^2 = \Delta\chi^2 \sum_{i,j} \left(\frac{\partial\Delta\sigma^{\pi^0}(p_T)}{\partial a_i}\right) H_{ij}^{-1} \left(\frac{\partial\Delta\sigma^{\pi^0}(p_T)}{\partial a_j}\right), \tag{3}$$

where a_i is a optimized parameter in the polarized PDFs. H_{ij} is the Hessian matrix which has the information of the parameter errors and the correlation between these parameters. The $\Delta\chi^2$ determines a confidence level of the uncertainty, and is estimated so that the level corresponds to the 1σ standard error. We choose the same value for the $\Delta\chi^2$ as the AAC analysis [2]. Further, the gradient terms for the cross section $\partial\Delta\sigma^{\pi^0}(p_T)/\partial a_i$ is given by

$$\begin{aligned}\frac{d\Delta\sigma^{\pi^0}}{dp_T} &= \sum_{a,b,c} \int_{\eta^{\min}}^{\eta^{\max}} d\eta \int_{x_a^{\min}}^{1} dx_a \int_{x_b^{\min}}^{1} dx_b \\ &\times \left[\frac{\partial\Delta f_a(x_a)}{\partial a_i}\Delta f_b(x_b) + \Delta f_a(x_a)\frac{\partial\Delta f_b(x_b)}{\partial a_i}\right] \\ &\times \mathcal{J}\left(\frac{\partial(\hat{t}, z)}{\partial(p_T, \eta)}\right) \frac{\Delta\hat{\sigma}^{ab\to cX}(\hat{s}, \hat{t})}{d\hat{t}} D_c^{\pi^0}(z), \end{aligned} \tag{4}$$

The gradient terms for the polarized PDF are analytically obtained at initial scale Q_0^2, and are numerically evolved to arbitrary scale Q^2 by the DGLAP equation.

In Fig. 3, the asymmetry uncertainty is compared to the statistical errors for the preliminary data by the PHENIX. We find that the uncertainty almost corresponds to the experimental errors, and is mainly composed by the uncertainty of $\Delta g(x)$. This fact indicates that the present π^0 data have the same constraint on the $\Delta g(x)$ as the polarized DIS data. At this stage, it is difficult to reduce the $\Delta g(x)$ uncertainty even if these data are included into the global analysis. Since the asymmetry uncertainty is very sensitive to the $\Delta g(x)$ uncertainty, the π^0 production has the potential to become a good probe for the $\Delta g(x)$ by future precise data.

In summary, we have investigated a correlation between the π^0 asymmetry and the configuration of $\Delta g(x)$. The preliminary data by the PHENIX indicates the large negative asymmetry at low p_T, which is inconsistent with the theoretical predictions by using the $\Delta g(x)$ from the polarized DIS data. Although the slight negative asymmetry can be obtained by modifying the $\Delta g(x)$, the functional form of $\Delta g(x)$ is required some restraints. Consequently, we exclude the possibility of a sizable negative value of the asymmetry at low p_T with the $\Delta g(x)$. Experimental uncertainties are however large at present. It is premature to conclude that the pQCD framework is not applicable to π^0 production in polarized pp collisions. Furthermore, we have indicated the existence of $\Delta g(x)$ which keeps the asymmetry to be negative in whole p_T regions. The negative asymmetry would suggest the negative polarization of $\Delta g(x)$. The PHENIX data motivate us to modify the functional form of $\Delta g(x)$ drastically. In future measurements, the asymmetry data in wide p_T region will provide useful information for clarifying the gluon spin content.

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